MA 303
Homework 1
(Homogeneous Linear Differential Equations)

Hoon Hong

1st-order, 1 variable

Problem:

$y' - 2y = 0$

$y(0) = -3$

1. Find the general solution:

$\lambda - 2 = 0$

$\lambda = 2$

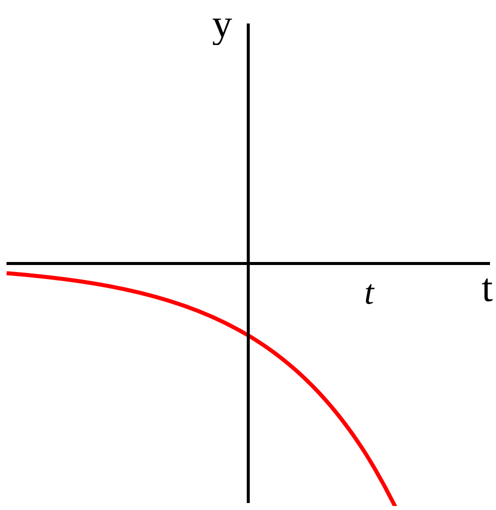
$y = C e^{2t}$

2. Find the particular solution:

$C = -3$

$y = -3 e^{2t}$

3. Sketch the particular solution:



Problem:

$y' - 2y = 0$

$y(0) = 4$

1. Find the general solution:

$\lambda - 2 = 0$

$\lambda = 2$

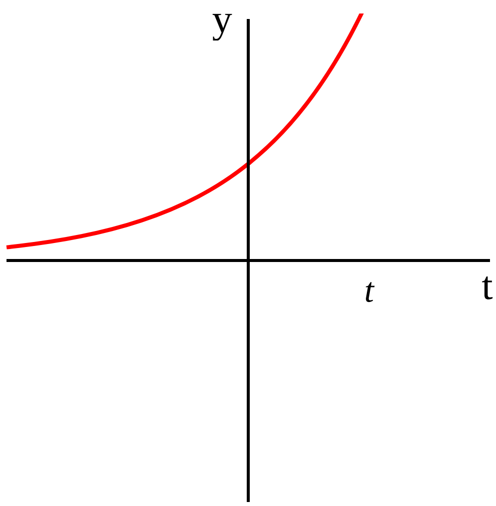
$y = C e^{2t}$

2. Find the particular solution:

$C = 4$

$y = 4 e^{2t}$

3. Sketch the particular solution:



Problem:

$$y' + 3y = 0$$

$$y(0) = 2$$

1. Find the general solution:

$$\lambda + 3 = 0$$

$$\lambda = -3$$

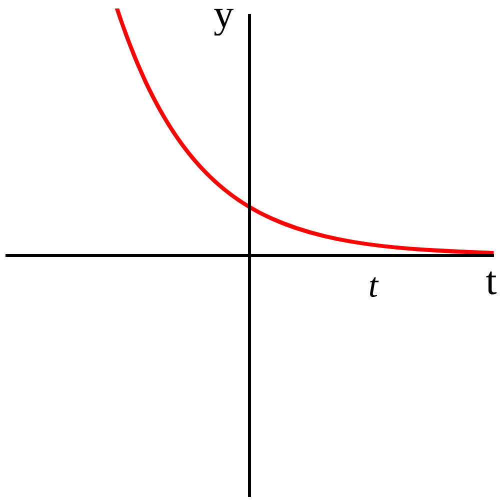
$$y = C e^{-3t}$$

2. Find the particular solution:

$$C = 2$$

$$y = 2 e^{-3t}$$

3. Sketch the particular solution:



Problem:

$y' + 3y = 0$

$y(0) = -2$

1. Find the general solution:

$\lambda + 3 = 0$

$\lambda = -3$

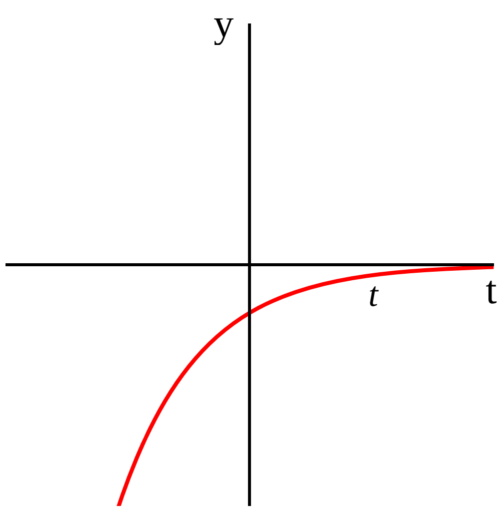
$y = C e^{-3t}$

2. Find the particular solution:

$C = -2$

$y = -2 e^{-3t}$

3. Sketch the particular solution:



2nd-order, 1 variable: Real eigenvalues

Problem:

$y'' + 3y' + 2y = 0$

$y(0) = -3$

$y'(0) = 2$

1. Find the general solution:

$\lambda^2 + 3\lambda + 2 = 0$

$\lambda = -1, -2$

$y = C_1 e^{-t} + C_2 e^{-2t}$

2. Find the particular solution:

$y' = -C_1 e^{-t} - 2C_2 e^{-2t}$

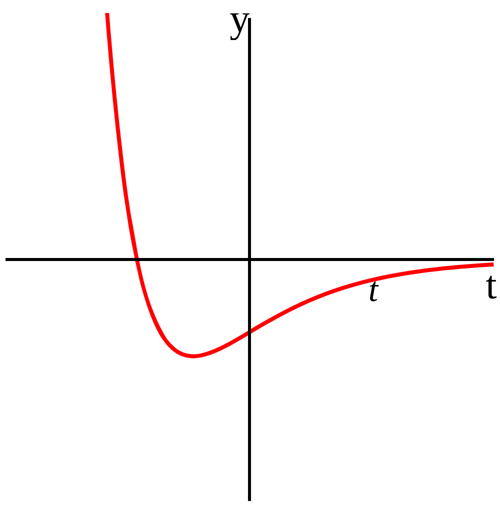
$C_1 + C_2 = -3$

$-C_1 - 2C_2 = 2$

$C_1 = -4, C_2 = 1$

$y = -4e^{-t} + e^{-2t}$

3. Sketch the particular solution:



Problem:

$y'' - 3y' + 2y = 0$

$y(0) = 3$

$y'(0) = 1$

1. Find the general solution:

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda = 2, 1$

$y = C_1 e^{2t} + C_2 e^{t}$

2. Find the particular solution:

$y' = 2C_1 e^{2t} + C_2 e^{t}$

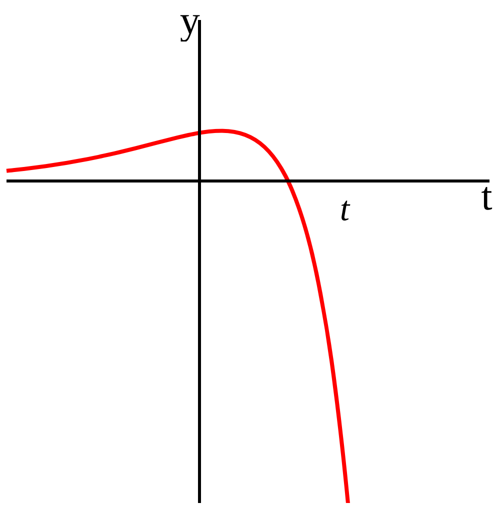
$C_1 + C_2 = 3$

$2C_1 + C_2 = 1$

$C_1 = -2, C_2 = 5$

$y = -2 e^{2t} + 5 e^{t}$

3. Sketch the particular solution:



Problem:

$y'' + y' - 6y = 0$

$y(0) = -3$

$y'(0) = 4$

1. Find the general solution:

$\lambda^2 + \lambda - 6 = 0$

$\lambda = 2, -3$

$y = C_1 e^{2t} + C_2 e^{-3t}$

2. Find the particular solution:

$y' = 2C_1 e^{2t} - 3C_2 e^{-3t}$

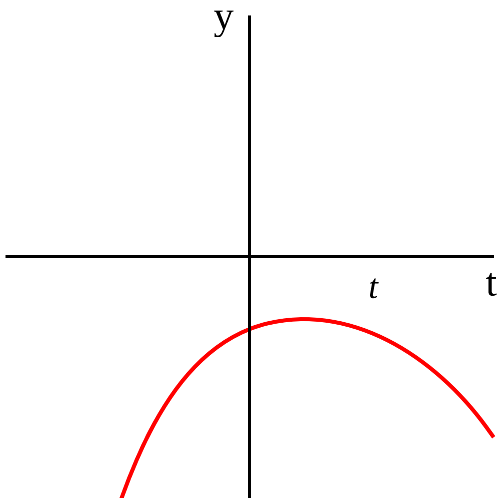
$C_1 + C_2 = -3$

$2C_1 - 3C_2 = 4$

$C_1 = -1, C_2 = -2$

$y = -e^{2t} - 2e^{-3t}$

3. Sketch the particular solution:



Problem:

$y'' + 5y' + 6y = 0$

$y(0) = 3$

$y'(0) = -8$

1. Find the general solution:

$\lambda^2 + 5\lambda + 6 = 0$

$\lambda = -2, -3$

$y = C_1 e^{-2t} + C_2 e^{-3t}$

2. Find the particular solution:

$y' = -2C_1 e^{-2t} - 3C_2 e^{-3t}$

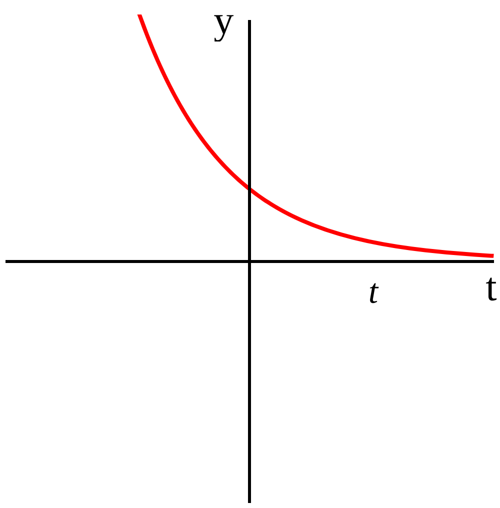
$C_1 + C_2 = 3$

$-2C_1 - 3C_2 = -8$

$C_1 = 1, C_2 = 2$

$y = e^{-2t} + 2e^{-3t}$

3. Sketch the particular solution:



Problem:

$$y'' + 7y' + 12y = 0$$

$$y(0) = 6$$

$$y'(0) = -22$$

1. Find the general solution:

$$\lambda^2 + 7\lambda + 12 = 0$$

$$\lambda = -3, -4$$

$$y = C_1 e^{-3t} + C_2 e^{-4t}$$

2. Find the particular solution:

$$y' = -3C_1 e^{-3t} - 4C_2 e^{-4t}$$

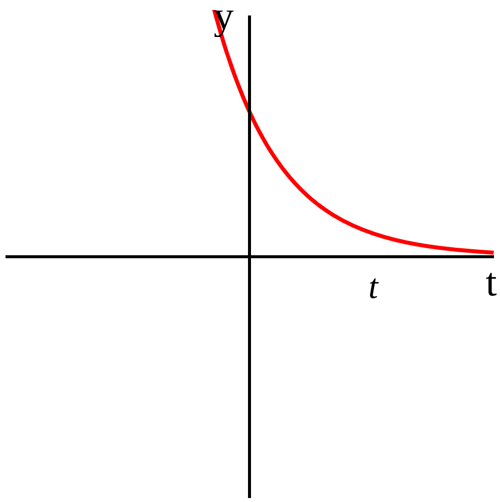
$$C_1 + C_2 = 6$$

$$-3C_1 - 4C_2 = -22$$

$$C_1 = 2, C_2 = 4$$

$$y = 2e^{-3t} + 4e^{-4t}$$

3. Sketch the particular solution:



Problem:

$$y'' - 5y' + 6y = 0$$

$$y(0) = 2$$

$$y'(0) = 1$$

1. Find the general solution:

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda = 3, 2$$

$$y = C_1 e^{3t} + C_2 e^{2t}$$

2. Find the particular solution:

$$y' = 3C_1 e^{3t} + 2C_2 e^{2t}$$

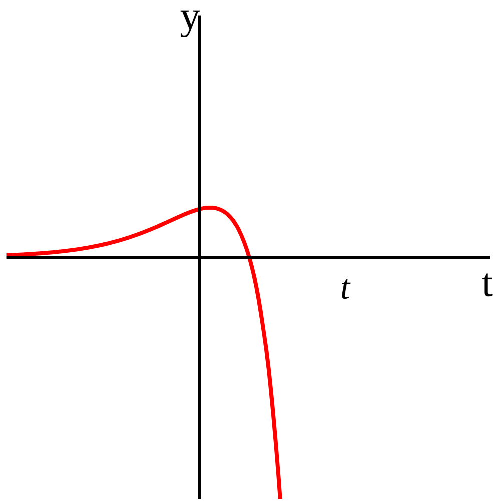
$$C_1 + C_2 = 2$$

$$3C_1 + 2C_2 = 1$$

$$C_1 = -3, C_2 = 5$$

$$y = -3e^{3t} + 5e^{2t}$$

3. Sketch the particular solution:



Problem:

$$y'' + y' - 2y = 0$$

$$y(0) = -4$$

$$y'(0) = 5$$

1. Find the general solution:

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda = 1, -2$$

$$y = C_1 e^t + C_2 e^{-2t}$$

2. Find the particular solution:

$$y' = C_1 e^t - 2C_2 e^{-2t}$$

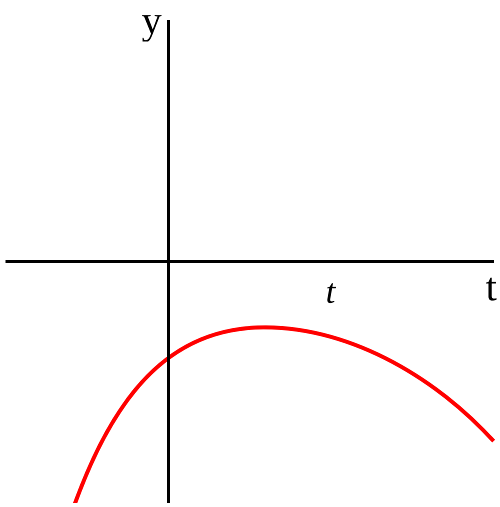
$$C_1 + C_2 = -4$$

$$C_1 - 2C_2 = 5$$

$$C_1 = -1, C_2 = -3$$

$$y = -e^t - 3e^{-2t}$$

3. Sketch the particular solution:



Problem:

$y'' + 5y' + 6y = 0$

$y(0) = -1$

$y'(0) = 1$

1. Find the general solution:

$\lambda^2 + 5\lambda + 6 = 0$

$\lambda = -2, -3$

$y = C_1 e^{-2t} + C_2 e^{-3t}$

2. Find the particular solution:

$y' = -2C_1 e^{-2t} - 3C_2 e^{-3t}$

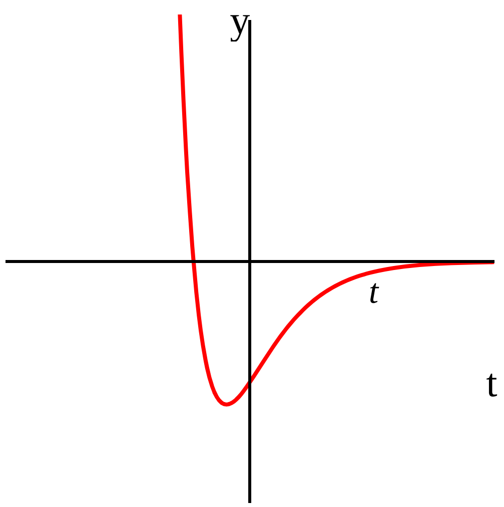
$C_1 + C_2 = -1$

$-2C_1 - 3C_2 = 1$

$C_1 = -2, C_2 = 1$

$y = -2e^{-2t} + e^{-3t}$

3. Sketch the particular solution:



▼ 2nd-order, 1 variable: Non-real eigenvalues

Problem:

$y'' - 4y' + 68y = 0$

$y(0) = 1$

$y'(0) = 10$

1. Find the general solution:

$\lambda^2 - 4\lambda + 68 = 0$

$\lambda = 2 + 8I, 2 - 8I$

$y = e^{2t}\left(C_1 \cos(8t) + C_2 \sin(8t)\right)$

2. Find the particular solution:

$y' = 2e^{2t}\left(C_1 \cos(8t) + C_2 \sin(8t)\right) + e^{2t}\left(-8C_1 \sin(8t) + 8C_2 \cos(8t)\right)$

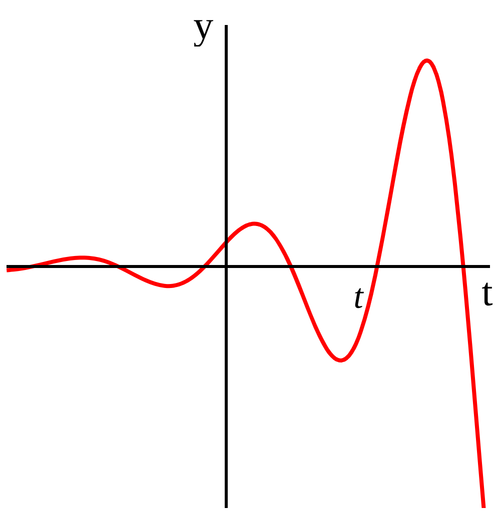
$C_1 = 1$

$2C_1 + 8C_2 = 10$

$C_2 = 1$

$y = e^{2t}(\cos(8t) + \sin(8t))$

3. Sketch the particular solution:



Problem:

$y'' + 2y' + 50y = 0$

$y(0) = 2$

$y'(0) = 19$

1. Find the general solution:

$\lambda^2 + 2\lambda + 50 = 0$

$\lambda = -1 + 7I, -1 - 7I$

$y = e^{-t}\left(C_1\cos(7t) + C_2\sin(7t)\right)$

2. Find the particular solution:

$y' = -e^{-t}\left(C_1\cos(7t) + C_2\sin(7t)\right) + e^{-t}\left(-7C_1\sin(7t) + 7C_2\cos(7t)\right)$

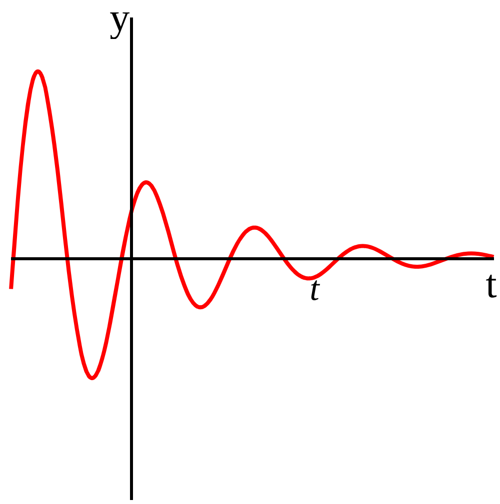
$C_1 = 2$

$-C_1 + 7C_2 = 19$

$C_2 = 3$

$y = e^{-t}(2\cos(7t) + 3\sin(7t))$

3. Sketch the particular solution:



Problem:

$y'' + 64\, y = 0$

$y(0) = 2$

$y'(0) = 8$

1. Find the general solution:

$\lambda^2 + 64 = 0$

$\lambda = 8\, I, -8\, I$

$y = C_1 \cos(8\, t) + C_2 \sin(8\, t)$

2. Find the particular solution:

$y' = -8\, C_1 \sin(8\, t) + 8\, C_2 \cos(8\, t)$

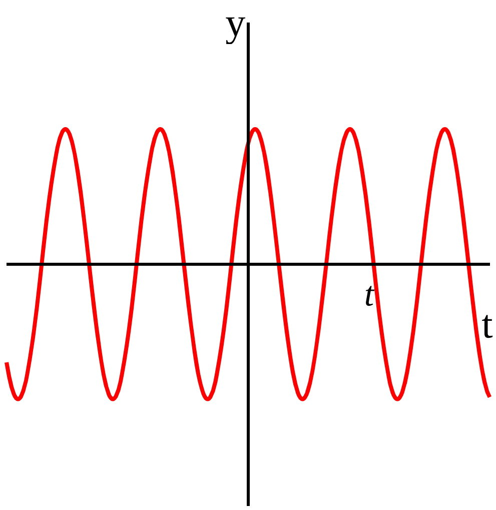
$C_1 = 2$

$8\, C_2 = 8$

$C_2 = 1$

$y = 2 \cos(8\, t) + \sin(8\, t)$

3. Sketch the particular solution:



Problem:

$y'' + 2\,y' + 82\,y = 0$

$y(0) = 0$

$y'(0) = 9$

1. Find the general solution:

$\lambda^2 + 2\,\lambda + 82 = 0$

$\lambda = -1 + 9\,I,\ -1 - 9\,I$

$y = e^{-t}\left(C_1 \cos(9\,t) + C_2 \sin(9\,t)\right)$

2. Find the particular solution:

$y' = -e^{-t}\left(C_1 \cos(9\,t) + C_2 \sin(9\,t)\right) + e^{-t}\left(-9\,C_1 \sin(9\,t) + 9\,C_2 \cos(9\,t)\right)$

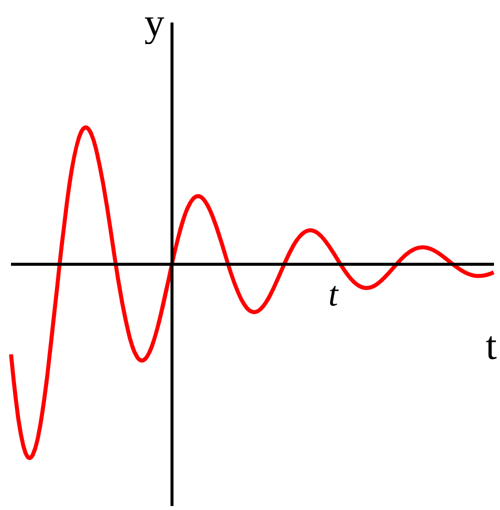
$C_1 = 0$

$-C_1 + 9\,C_2 = 9$

$C_2 = 1$

$y = e^{-t} \sin(9\,t)$

3. Sketch the particular solution:



Problem:

$$y'' - 4y' + 125y = 0$$

$$y(0) = 1$$

$$y'(0) = 35$$

1. Find the general solution:

$$\lambda^2 - 4\lambda + 125 = 0$$

$$\lambda = 2 + 11\,I,\ 2 - 11\,I$$

$$y = e^{2t}\left(C_1 \cos(11t) + C_2 \sin(11t)\right)$$

2. Find the particular solution:

$$y' = 2e^{2t}\left(C_1 \cos(11t) + C_2 \sin(11t)\right) + e^{2t}\left(-11 C_1 \sin(11t) + 11 C_2 \cos(11t)\right)$$

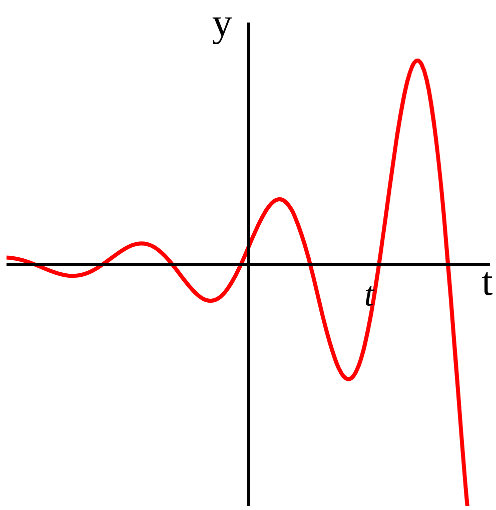
$$C_1 = 1$$

$$2C_1 + 11C_2 = 35$$

$$C_2 = 3$$

$$y = e^{2t}(\cos(11t) + 3\sin(11t))$$

3. Sketch the particular solution:



Problem:

$y'' + 9y = 0$

$y(0) = 1$

$y'(0) = -3$

1. Find the general solution:

$\lambda^2 + 9 = 0$

$\lambda = 3I, -3I$

$y = C_1 \cos(3t) + C_2 \sin(3t)$

2. Find the particular solution:

$y' = -3C_1 \sin(3t) + 3C_2 \cos(3t)$

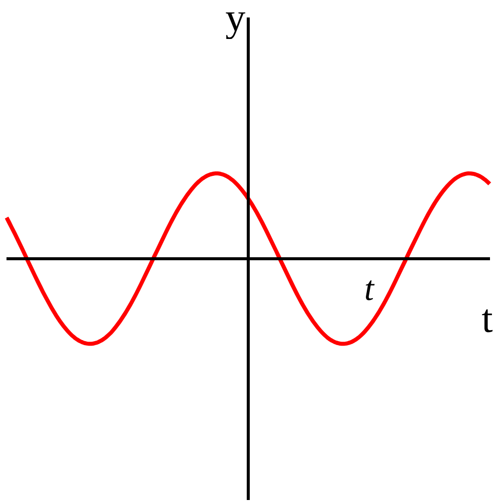
$C_1 = 1$

$3C_2 = -3$

$C_2 = -1$

$y = \cos(3t) - \sin(3t)$

3. Sketch the particular solution:



1st-order, 2 variables: Introduction

Problem:

$y1' = -y1 + 2\, y2$

$y2' = 2\, y1 + 2\, y2$

$y1(0) = 3$

$y2(0) = 1$

1. Find the general solution:

$(-1 - \lambda)\, v_1 + 2\, v_2 = 0$

$2\, v_1 + (2 - \lambda)\, v_2 = 0$

$\lambda^2 - \lambda - 6 = 0$

$\lambda = 3$

$v_1 = 2\, C_1$

$v_2 = 4\, C_1$

$\lambda = -2$

$v_1 = 2\, C_2$

$v_2 = -C_2$

$y1 = 2\, C_1\, e^{3\,t} + 2\, C_2\, e^{-2\,t}$

$y2 = 4\, C_1\, e^{3\,t} - C_2\, e^{-2\,t}$

2. Find the particular solution:

$2\, C_1 + 2\, C_2 = 3$

$4\, C_1 - C_2 = 1$

$C_1 = \frac{1}{2}, C_2 = 1$

$y1 = e^{3\,t} + 2\, e^{-2\,t}$

$y2 = 2\, e^{3\,t} - e^{-2\,t}$

Problem:

$$y1' = y1 - 2\,y2$$

$$y2' = -2\,y1 - 2\,y2$$

$$y1(0) = 3$$

$$y2(0) = 1$$

1. Find the general solution:

$$(1 - \lambda)\,v_1 - 2\,v_2 = 0$$

$$-2\,v_1 + (-2 - \lambda)\,v_2 = 0$$

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = -3$$

$$v_1 = -2\,C_1$$

$$v_2 = -4\,C_1$$

$$\lambda = 2$$

$$v_1 = -2\,C_2$$

$$v_2 = C_2$$

$$y1 = -2\,C_1\,e^{-3t} - 2\,C_2\,e^{2t}$$

$$y2 = -4\,C_1\,e^{-3t} + C_2\,e^{2t}$$

2. Find the particular solution:

$$-2\,C_1 - 2\,C_2 = 3$$

$$-4\,C_1 + C_2 = 1$$

$$C_1 = -\frac{1}{2},\ C_2 = -1$$

$$y1 = e^{-3t} + 2\,e^{2t}$$

$$y2 = 2\,e^{-3t} - e^{2t}$$

Problem:

$y1' = -5\, y1 - 2\, y2$

$y2' = 2\, y1$

$y1(0) = 1$

$y2(0) = 1$

1. Find the general solution:

$(-5 - \lambda)\, v_1 - 2\, v_2 = 0$

$-\lambda\, v_2 + 2\, v_1 = 0$

$\lambda^2 + 5\, \lambda + 4 = 0$

$\lambda = -1$

$v_1 = -2\, C_1$

$v_2 = 4\, C_1$

$\lambda = -4$

$v_1 = -2\, C_2$

$v_2 = C_2$

$y1 = -2\, C_1\, e^{-t} - 2\, C_2\, e^{-4t}$

$y2 = 4\, C_1\, e^{-t} + C_2\, e^{-4t}$

2. Find the particular solution:

$-2\, C_1 - 2\, C_2 = 1$

$4\, C_1 + C_2 = 1$

$C_1 = \frac{1}{2},\, C_2 = -1$

$y1 = -e^{-t} + 2\, e^{-4t}$

$y2 = 2\, e^{-t} - e^{-4t}$

Problem:

$y1' = 3\,y1 - y2$

$y2' = -2\,y1 + 4\,y2$

$y1(0) = 1$

$y2(0) = 4$

1. Find the general solution:

$(3 - \lambda)\,v_1 - v_2 = 0$

$-2\,v_1 + (4 - \lambda)\,v_2 = 0$

$\lambda^2 - 7\,\lambda + 10 = 0$

$\lambda = 2$

$v_1 = -C_1$

$v_2 = -C_1$

$\lambda = 5$

$v_1 = -C_2$

$v_2 = 2\,C_2$

$y1 = -C_1\,e^{2t} - C_2\,e^{5t}$

$y2 = -C_1\,e^{2t} + 2\,C_2\,e^{5t}$

2. Find the particular solution:

$-C_1 - C_2 = 1$

$-C_1 + 2\,C_2 = 4$

$C_1 = -2,\ C_2 = 1$

$y1 = 2\,e^{2t} - e^{5t}$

$y2 = 2\,e^{2t} + 2\,e^{5t}$

1st-order, 2 variables: Real eigenvalues. Opposite Signs

Problem:

$$y' = \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} -1-\lambda & 2 \\ 2 & 2-\lambda \end{bmatrix} = 0$$

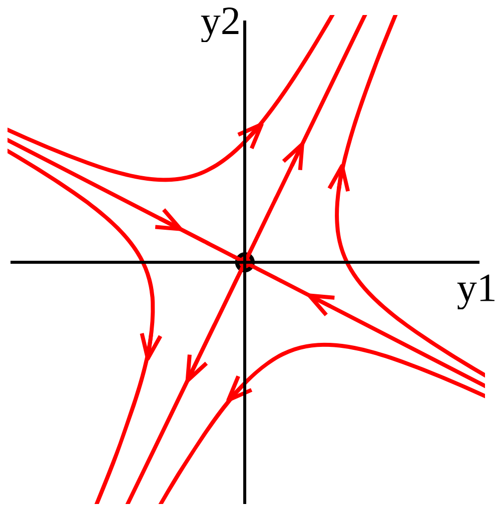
$$\lambda^2 - \lambda - 6 = 0$$

$$\lambda = 3, v = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\lambda = -2, v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t}$$

2. Sketch the general solution:



Saddle

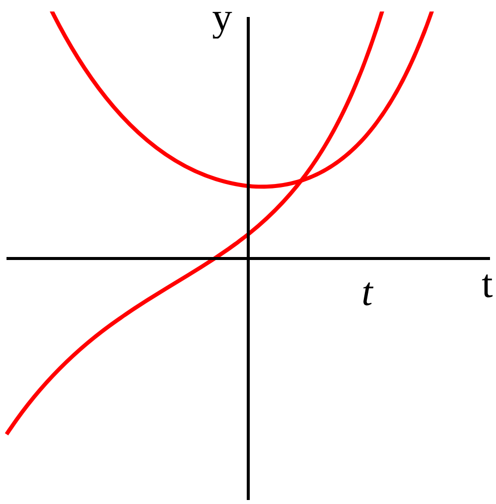
3. Find the particular solution:

$$C_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$C_1 = \frac{1}{2}, C_2 = 1$$

$$y = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 1 & -2 \\ -2 & -2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} 1-\lambda & -2 \\ -2 & -2-\lambda \end{bmatrix} = 0$$

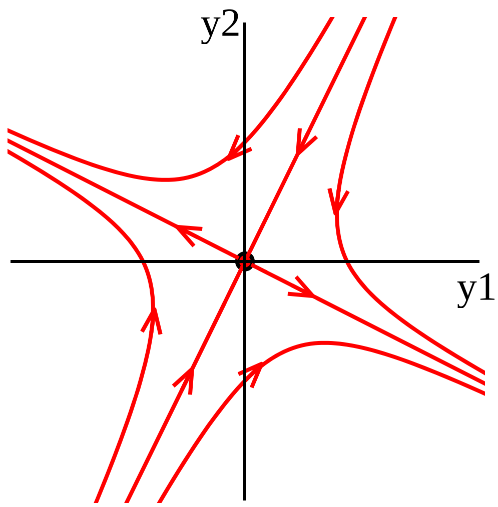
$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = -3,\ v = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$\lambda = 2,\ v = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -2 \\ -4 \end{bmatrix} e^{-3t} + C_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{2t}$$

2. Sketch the general solution:



Saddle

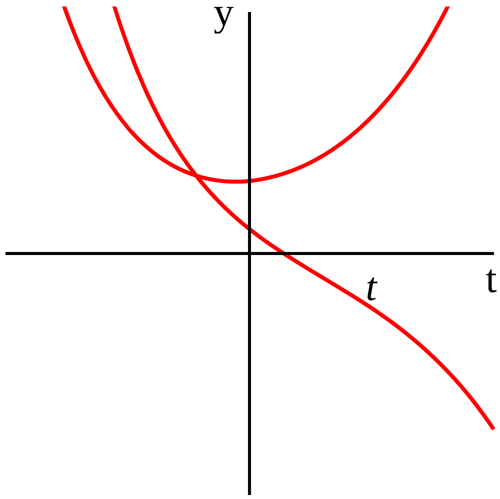
3. Find the particular solution:

$$C_1 \begin{bmatrix} -2 \\ -4 \end{bmatrix} + C_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$C_1 = -\frac{1}{2},\ C_2 = -1$$

$$y = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-3t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} -2-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix} = 0$$

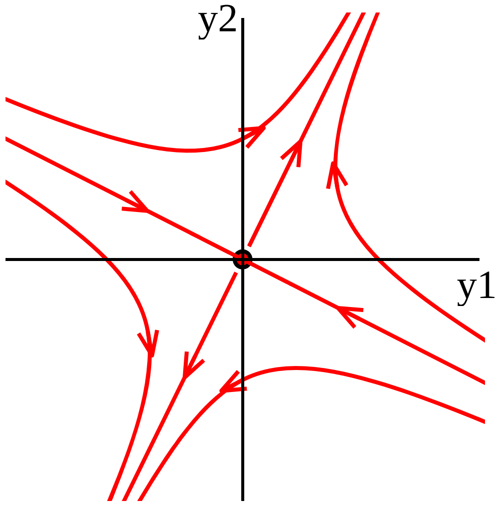
$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = 2, v = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\lambda = -3, v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{2t} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-3t}$$

2. Sketch the general solution:



Saddle

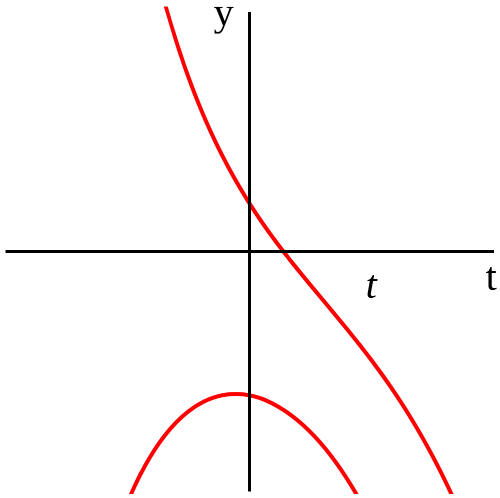
3. Find the particular solution:

$$C_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$C_1 = -1, C_2 = 2$$

$$y = \begin{bmatrix} -2 \\ -4 \end{bmatrix} e^{2t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-3t}$$

4. Sketch the particular solution:



1st-order, 2 variables: Real eigenvalues. Same Signs

Problem:

$$y' = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} 3-\lambda & -1 \\ -2 & 4-\lambda \end{bmatrix} = 0$$

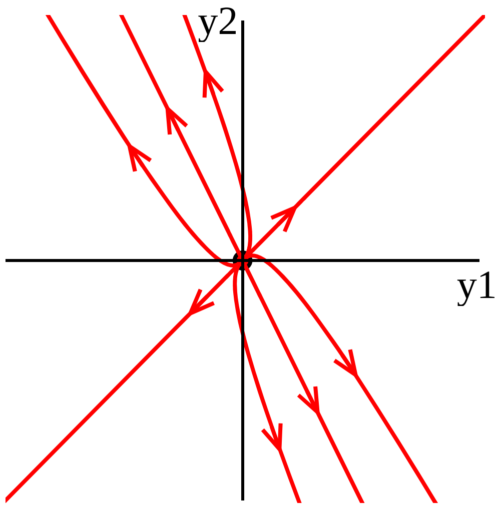
$$\lambda^2 - 7\lambda + 10 = 0$$

$$\lambda = 2, v = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\lambda = 5, v = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{2t} + C_2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{5t}$$

2. Sketch the general solution:



Unstable

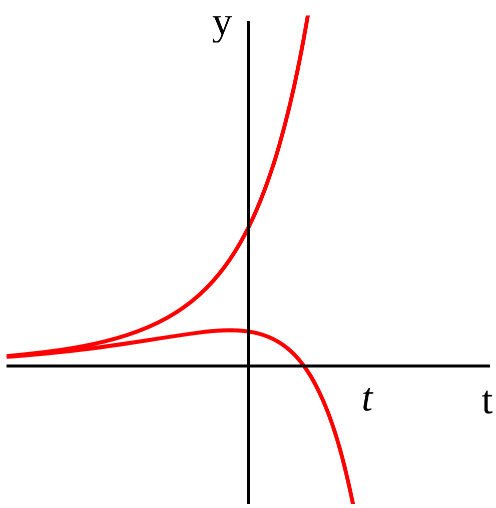
3. Find the particular solution:

$$C_1\begin{bmatrix}-1\\-1\end{bmatrix}+C_2\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}1\\4\end{bmatrix}$$

$C_1=-2, C_2=1$

$$y=\begin{bmatrix}2\\2\end{bmatrix}e^{2t}+\begin{bmatrix}-1\\2\end{bmatrix}e^{5t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} -3-\lambda & 2 \\ 1 & -2-\lambda \end{bmatrix} = 0$$

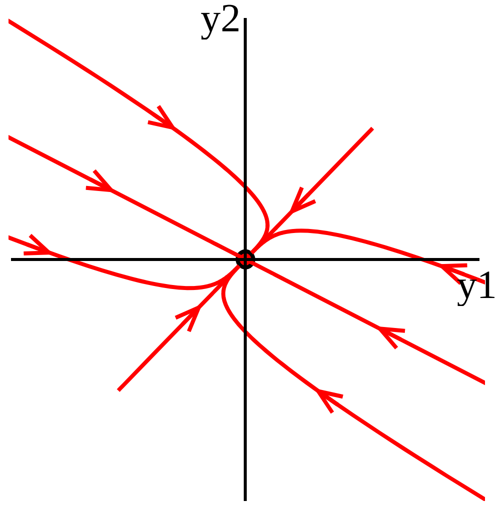
$$\lambda^2 + 5\lambda + 4 = 0$$

$$\lambda = -1, v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\lambda = -4, v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{-t} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-4t}$$

2. Sketch the general solution:



Stable

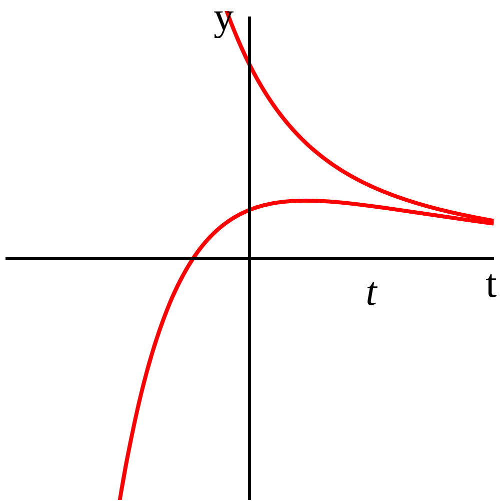
3. Find the particular solution:

$$C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

$$C_1 = 2, C_2 = 2$$

$$y = \begin{bmatrix} 4 \\ 4 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-4t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} 2-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix} = 0$$

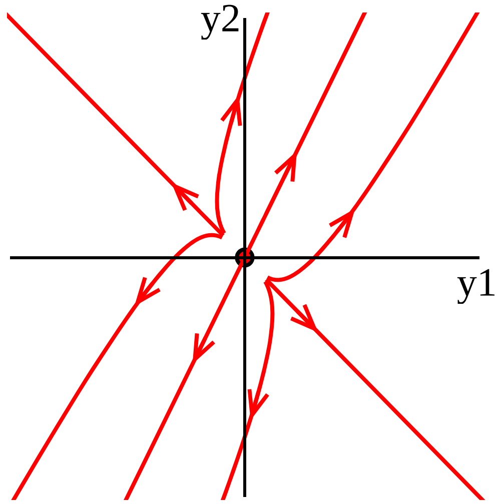
$$\lambda^2 - 5\lambda + 4 = 0$$

$$\lambda = 1, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = 4, v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + C_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{4t}$$

2. Sketch the general solution:



Unstable

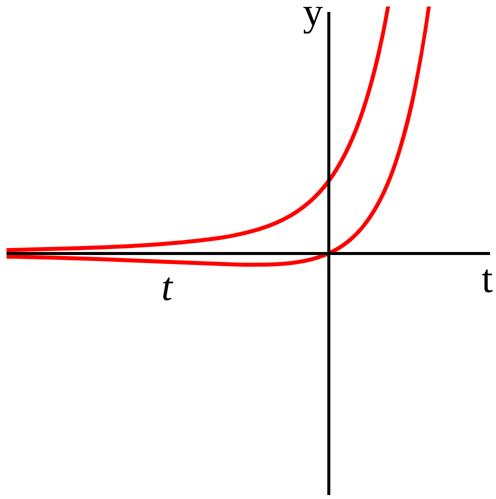
3. Find the particular solution:

$$C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$C_1 = -1, C_2 = 1$$

$$y = \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{t} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{4t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

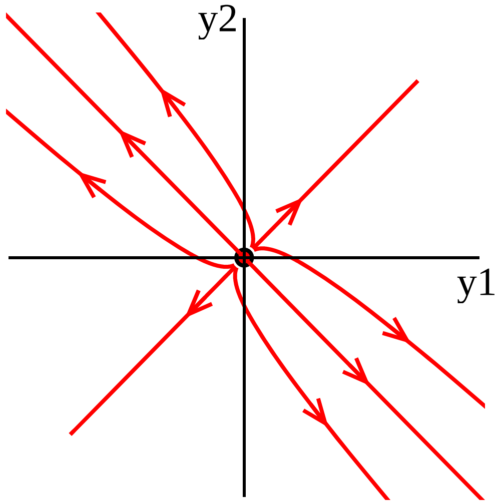
$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda = 1, v = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\lambda = 3, v = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{t} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{3t}$$

2. Sketch the general solution:



Unstable

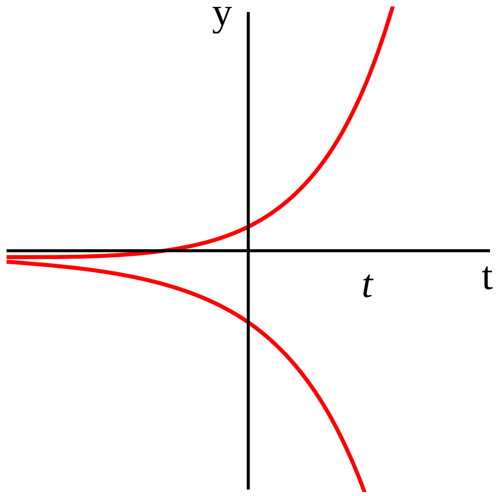
3. Find the particular solution:

$$C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$C_1 = 1, C_2 = 2$$

$$y = \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{t} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} e^{3t}$$

4. Sketch the particular solution:



1st-order 2 variables: Non-real eigenvalues

Problem:

$$y' = \begin{bmatrix} -2 & -10 \\ 10 & -2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} -2-\lambda & -10 \\ 10 & -2-\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 4\lambda + 104 = 0$$

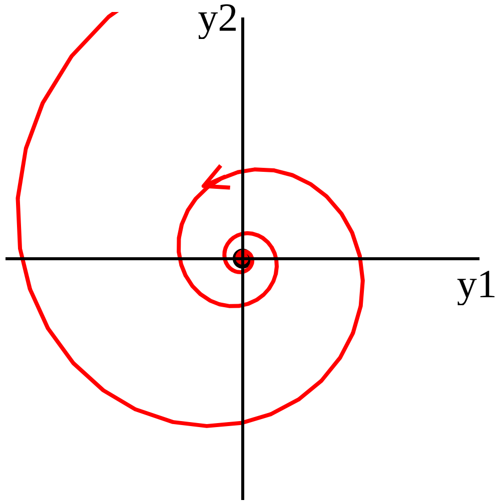
$$\lambda = -2 + 10\,\mathrm{I},\ v = \begin{bmatrix} -10 \\ 10\,\mathrm{I} \end{bmatrix}$$

$$\lambda = -2 - 10\,\mathrm{I},\ v = \begin{bmatrix} -10 \\ -10\,\mathrm{I} \end{bmatrix}$$

$$\alpha = -2,\ \beta = 10,\ p = \begin{bmatrix} -10 \\ 0 \end{bmatrix},\ q = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$y = \mathrm{e}^{-2t}\left(\left(C_1 \begin{bmatrix} -10 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 10 \end{bmatrix}\right)\cos(10\,t) + \left(C_2 \begin{bmatrix} -10 \\ 0 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 10 \end{bmatrix}\right)\sin(10\,t)\right)$$

2. Sketch the general solution:



Stable

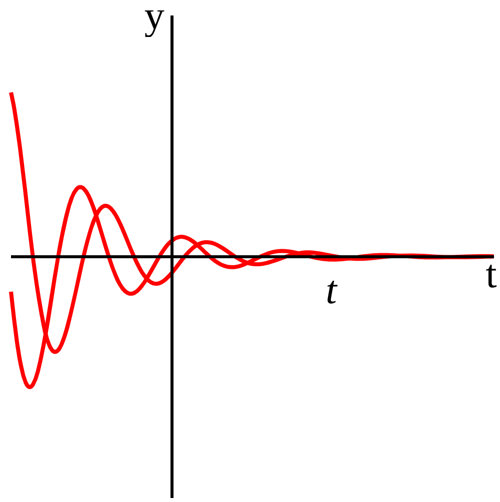
3. Find the particular solution:

$$C_1 \begin{bmatrix} -10 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C_1 = -\frac{1}{10}, C_2 = -\frac{1}{10}$$

$$y = e^{-2t} \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \cos(10\,t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin(10\,t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -7 & 16 \\ -8 & 9 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} -7-\lambda & 16 \\ -8 & 9-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - 2\lambda + 65 = 0$$

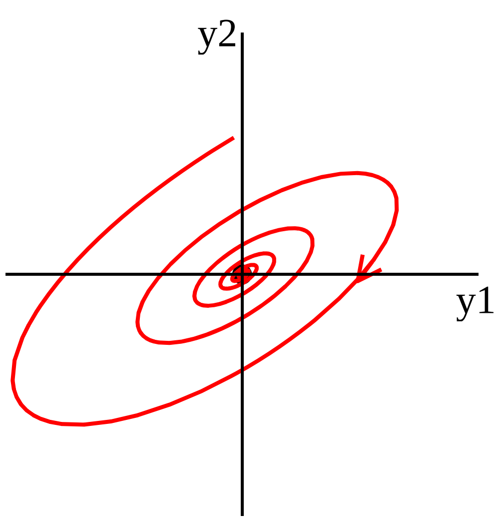
$$\lambda = 1 + 8\,\mathrm{I},\ v = \begin{bmatrix} 16 \\ 8 + 8\,\mathrm{I} \end{bmatrix}$$

$$\lambda = 1 - 8\,\mathrm{I},\ v = \begin{bmatrix} 16 \\ 8 - 8\,\mathrm{I} \end{bmatrix}$$

$$\alpha = 1,\ \beta = 8,\ p = \begin{bmatrix} 16 \\ 8 \end{bmatrix},\ q = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$$

$$y = e^{t}\left(\left(C_1 \begin{bmatrix} 16 \\ 8 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 8 \end{bmatrix}\right)\cos(8\,t) + \left(C_2 \begin{bmatrix} 16 \\ 8 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 8 \end{bmatrix}\right)\sin(8\,t)\right)$$

2. Sketch the general solution:



Unstable

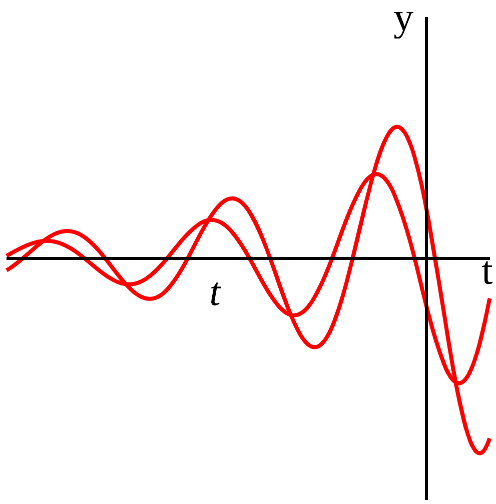
3. Find the particular solution:

$$C_1\begin{bmatrix}16\\8\end{bmatrix}+C_2\begin{bmatrix}0\\8\end{bmatrix}=\begin{bmatrix}4\\-4\end{bmatrix}$$

$$C_1=\frac{1}{4},\ C_2=-\frac{3}{4}$$

$$y=e^{t}\left(\begin{bmatrix}4\\-4\end{bmatrix}\cos(8\,t)+\begin{bmatrix}-12\\-8\end{bmatrix}\sin(8\,t)\right)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 2 & 10 \\ -4 & -2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} 2-\lambda & 10 \\ -4 & -2-\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 36 = 0$$

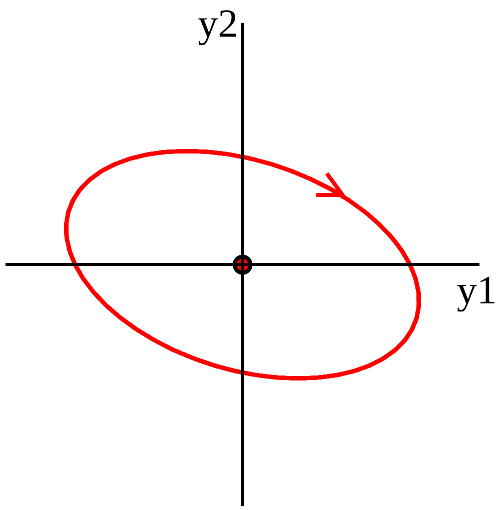
$$\lambda = 6\,I,\ v = \begin{bmatrix} 10 \\ -2+6\,I \end{bmatrix}$$

$$\lambda = -6\,I,\ v = \begin{bmatrix} 10 \\ -2-6\,I \end{bmatrix}$$

$$\alpha = 0,\ \beta = 6,\ p = \begin{bmatrix} 10 \\ -2 \end{bmatrix},\ q = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

$$y = \left(C_1 \begin{bmatrix} 10 \\ -2 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 6 \end{bmatrix} \right) \cos(6\,t) + \left(C_2 \begin{bmatrix} 10 \\ -2 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 6 \end{bmatrix} \right) \sin(6\,t)$$

2. Sketch the general solution:



center

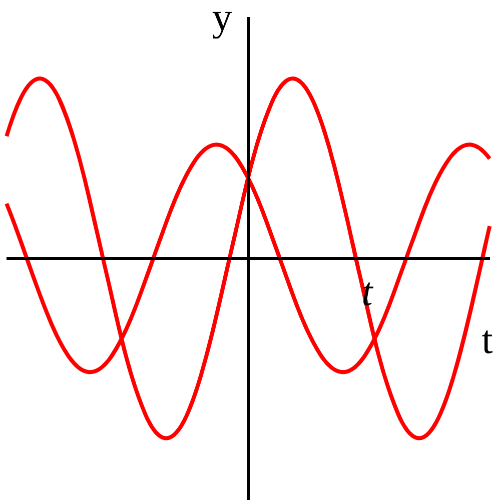
3. Find the particular solution:

$$C_1 \begin{bmatrix} 10 \\ -2 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C_1 = \frac{1}{10}, C_2 = \frac{1}{5}$$

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cos(6\,t) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} \sin(6\,t)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} 1-\lambda & -4 \\ 4 & 1-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - 2\lambda + 17 = 0$$

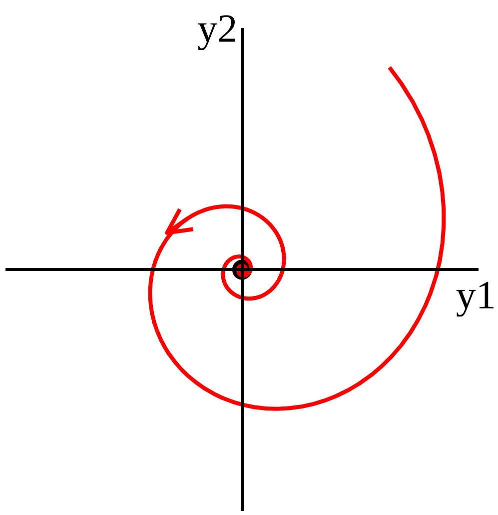
$$\lambda = 1 + 4\,\mathrm{I},\ v = \begin{bmatrix} -4 \\ 4\,\mathrm{I} \end{bmatrix}$$

$$\lambda = 1 - 4\,\mathrm{I},\ v = \begin{bmatrix} -4 \\ -4\,\mathrm{I} \end{bmatrix}$$

$$\alpha = 1, \beta = 4, p = \begin{bmatrix} -4 \\ 0 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$y = \mathrm{e}^t \left(\left(C_1 \begin{bmatrix} -4 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 4 \end{bmatrix} \right) \cos(4\,t) + \left(C_2 \begin{bmatrix} -4 \\ 0 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 4 \end{bmatrix} \right) \sin(4\,t) \right)$$

2. Sketch the general solution:



Unstable

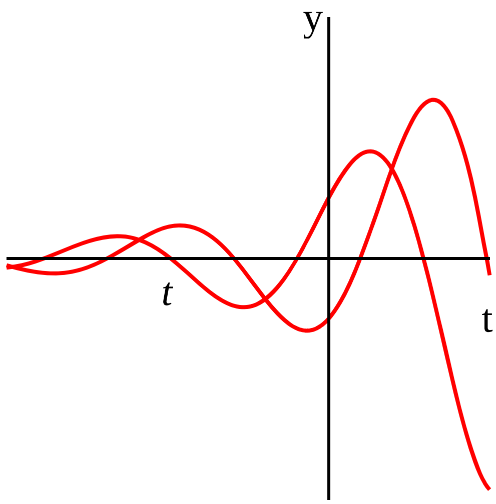
3. Find the particular solution:

$$C_1 \begin{bmatrix} -4 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C_1 = -\frac{1}{4}, C_2 = -\frac{1}{4}$$

$$y = e^t \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \cos(4t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin(4t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -14 & 24 \\ -12 & 10 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} -14-\lambda & 24 \\ -12 & 10-\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 4\lambda + 148 = 0$$

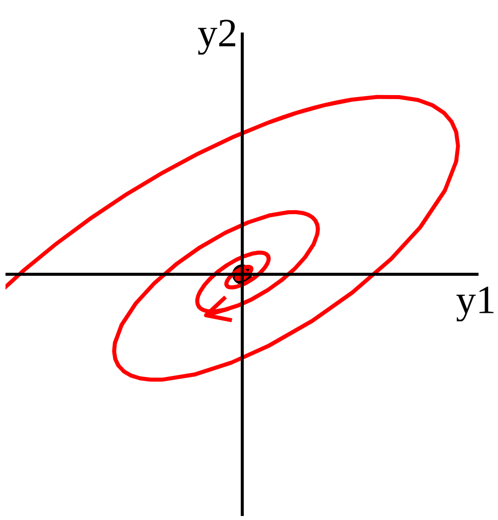
$$\lambda = -2 + 12\,I,\ v = \begin{bmatrix} 24 \\ 12 + 12\,I \end{bmatrix}$$

$$\lambda = -2 - 12\,I,\ v = \begin{bmatrix} 24 \\ 12 - 12\,I \end{bmatrix}$$

$$\alpha = -2,\ \beta = 12,\ p = \begin{bmatrix} 24 \\ 12 \end{bmatrix},\ q = \begin{bmatrix} 0 \\ 12 \end{bmatrix}$$

$$y = e^{-2t}\left(\left(C_1 \begin{bmatrix} 24 \\ 12 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 12 \end{bmatrix}\right)\cos(12\,t) + \left(C_2 \begin{bmatrix} 24 \\ 12 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 12 \end{bmatrix}\right)\sin(12\,t)\right)$$

2. Sketch the general solution:



Stable

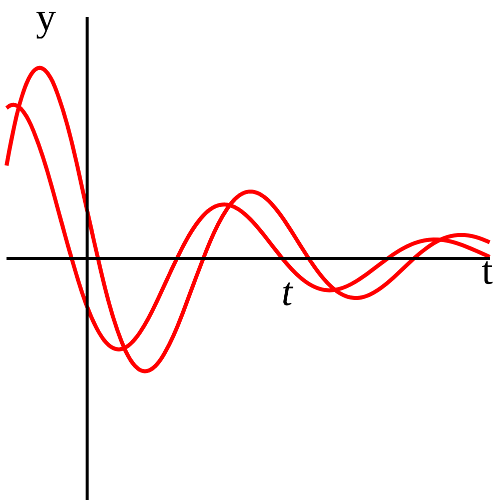
3. Find the particular solution:

$$C_1 \begin{bmatrix} 24 \\ 12 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 12 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$C_1 = \frac{1}{6}, C_2 = -\frac{1}{2}$$

$$y = e^{-2t} \left(\begin{bmatrix} 4 \\ -4 \end{bmatrix} \cos(12t) + \begin{bmatrix} -12 \\ -8 \end{bmatrix} \sin(12t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 6 & -10 \\ 4 & -6 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$Det \begin{bmatrix} 6-\lambda & -10 \\ 4 & -6-\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 4 = 0$$

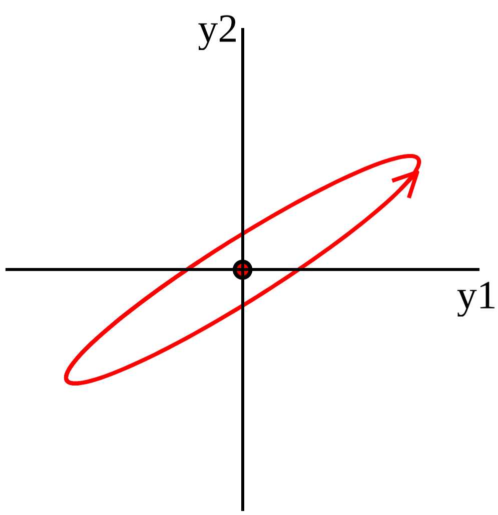
$$\lambda = 2\,\mathrm{I},\ v = \begin{bmatrix} -10 \\ -6+2\,\mathrm{I} \end{bmatrix}$$

$$\lambda = -2\,\mathrm{I},\ v = \begin{bmatrix} -10 \\ -6-2\,\mathrm{I} \end{bmatrix}$$

$$\alpha = 0,\ \beta = 2,\ p = \begin{bmatrix} -10 \\ -6 \end{bmatrix},\ q = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$y = \left(C_1 \begin{bmatrix} -10 \\ -6 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right) \cos(2\,t) + \left(C_2 \begin{bmatrix} -10 \\ -6 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right) \sin(2\,t)$$

2. Sketch the general solution:



center

3. Find the particular solution:

$$C_1 \begin{bmatrix} -10 \\ -6 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C_1 = -\frac{1}{5}, C_2 = -\frac{1}{10}$$

$$y = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cos(2\,t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin(2\,t)$$

4. Sketch the particular solution: